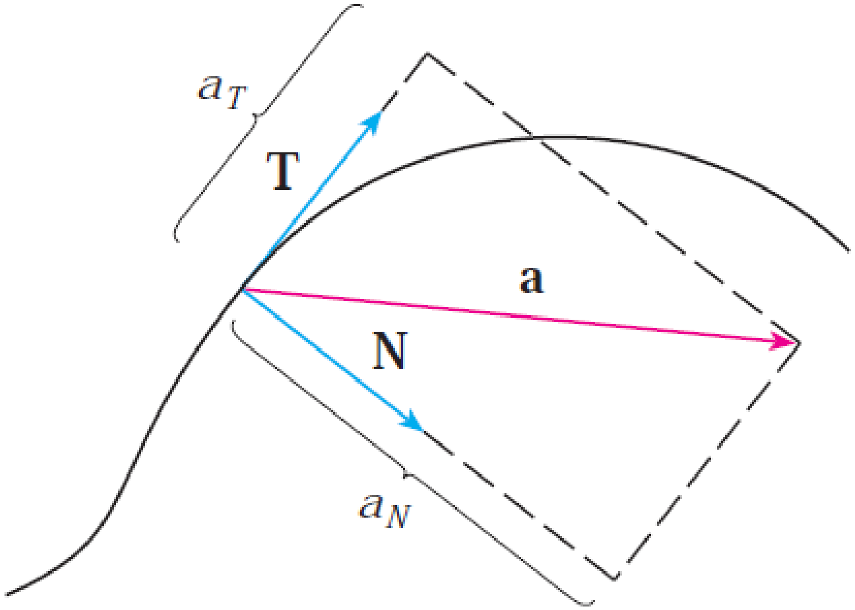
Close Tue: 10.3
Close Thu: 14.1, 14.3 (part 1)
Exam 1 will be returned Tuesday.
No office hours today (grading day).
Today: Finish 13.4 and start 10.3

Finishing 13.4 Acceleration/Velocity

Entry Task: A ball with mass $m = 0.8$ kg is thrown northward into the air with initial speed of 30 m/sec at an angle of 30 degrees with the ground. A west wind applies a steady force of 4 N on the ball (west to east).

If you are standing on level ground, where does the ball land?

Measuring and describing acceleration



Recall: $\text{comp}_{\boldsymbol{b}}(\boldsymbol{a}) = \frac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{b}|}$ = length.

We define the tangential and normal components of acceleration by:

$a_T = \text{comp}_{\boldsymbol{T}}(\boldsymbol{a}) = \boldsymbol{a}\cdot\boldsymbol{T}$ = tangential

$a_N = \text{comp}_{\boldsymbol{N}}(\boldsymbol{a}) = \boldsymbol{a}\cdot\boldsymbol{N}$ = normal

For computing use,

$$a_T = \frac{\vec{\boldsymbol{r}}'\cdot\vec{\boldsymbol{r}}''}{|\vec{\boldsymbol{r}}'|} \quad \text{and} \quad a_T = \frac{|\vec{\boldsymbol{r}}'\times\vec{\boldsymbol{r}}''|}{|\vec{\boldsymbol{r}}'|}$$

For interpreting use,

$a_T = v' = \frac{d}{dt}|r'(t)|$ = "deriv. of speed"

$a_N = kv^2$ = curvature · (speed)^2

Example:

$$\vec{\boldsymbol{r}}(t) = < \cos(t), \sin(t), t >$$

Find the tangential and normal components of acceleration.

Deriving interpretations:

Note that: $\boldsymbol{a} = a_T\boldsymbol{T} + a_N\boldsymbol{N}$

Let $v(t) = |\vec{\boldsymbol{v}}(t)|$ = speed.

1. $\vec{\boldsymbol{T}}(t) = \dfrac{\vec{\boldsymbol{r}}'(t)}{|\vec{\boldsymbol{r}}'(t)|} = \dfrac{\vec{\boldsymbol{v}}(t)}{v(t)}$ implies $\vec{\boldsymbol{v}} = v\vec{\boldsymbol{T}}$.

2. $\kappa(t) = \dfrac{|\vec{\boldsymbol{T}}'(t)|}{|\vec{\boldsymbol{r}}'(t)|} = \dfrac{|\vec{\boldsymbol{T}}'|}{v(t)}$ implies $|\vec{\boldsymbol{T}}'| = \kappa v$.

3. $\vec{\boldsymbol{N}}(t) = \dfrac{\vec{\boldsymbol{T}}'(t)}{|\vec{\boldsymbol{T}}'(t)|} = \dfrac{\vec{\boldsymbol{T}}'}{\kappa v}$ implies $\vec{\boldsymbol{T}}' = \kappa v\vec{\boldsymbol{N}}$.

Differentiating the first fact above gives

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = v'\vec{\boldsymbol{T}} + v\vec{\boldsymbol{T}}', \text{ so}$$

$$\vec{\boldsymbol{a}} = \vec{\boldsymbol{v}}' = v'\vec{\boldsymbol{T}} + kv^2\vec{\boldsymbol{N}}.$$

Conclusion:

$a_T = v' = \dfrac{d}{dt}|r'(t)| =$ “deriv. of speed”

$a_N = kv^2 =$ curvature · (speed)^2

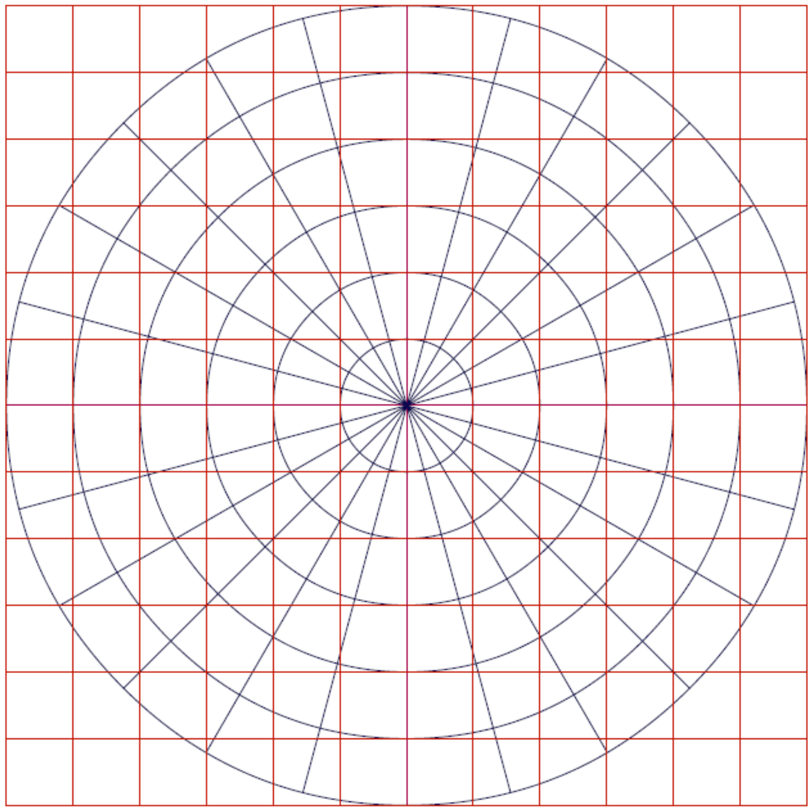
10.3 Polar Coordinates

Goal: A 2D coordinate system good for circular/arcing paths.

Cartesian	Polar
Given (x, y) 1. Stand at origin.	Given (r, θ) 1. Stand at origin facing the positive *x*-axis.
2. Move x-units on x-axis. pos. = right, neg. = left	2. Rotate by θ. pos. = ccw, neg. = clockwise
3. Move y-units parallel to y-axis. pos. = up neg. = down	3. Walk *r*-units in direction you are facing. pos. = forward neg. = backward

Example: Plot these polar points

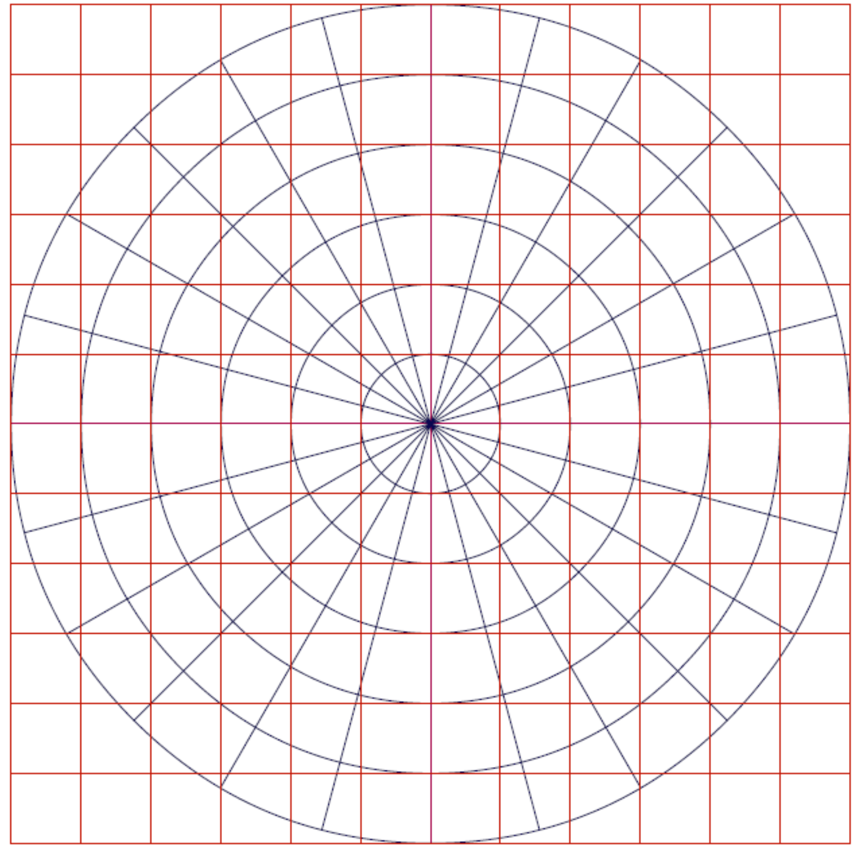
1. $(r, \theta) = (1, \pi/2)$
2. $(r, \theta) = (3, 5\pi/4)$
3. $(r, \theta) = (0, \pi/3)$
4. $(r, \theta) = (-1, 3\pi/2)$
5. $(r, \theta) = (4, 0)$
6. $(r, \theta) = (4, 100\ \pi)$



From trig we already know:

$$x = r\cos(\theta), \qquad y = r\sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \qquad x^2 + y^2 = r^2$$



Exercise:

1. Describe all pts where $r = 3$.

2. Describe all pts where $\theta = \pi/4$.

Plotting Polar Curves

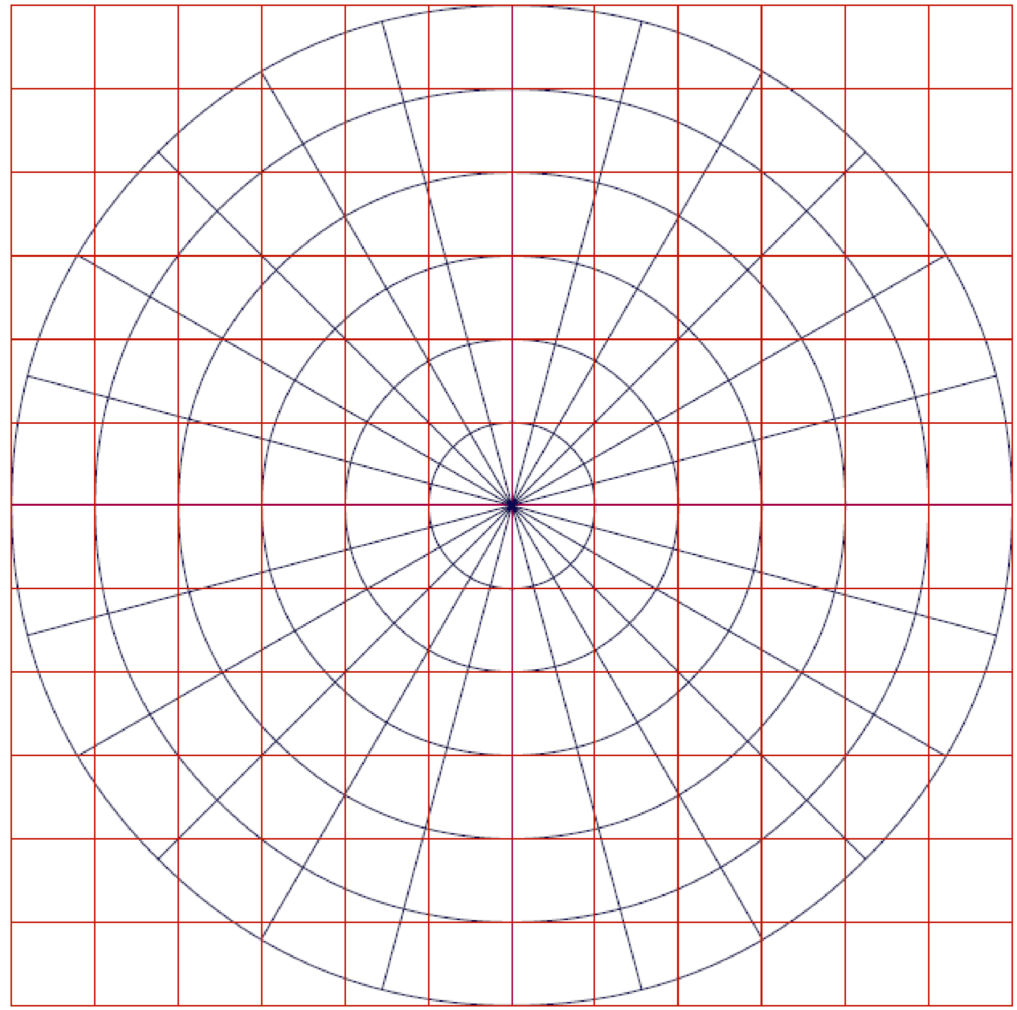
Option 1: Try to convert to x and y.
Then hope you recognize the curve.

Option 2: **Plot points!**
Start with 0, π/2, π, 3π/2 (intercepts).
For more detail do multiples of π/6
and π/4.

Example: Graph r = sin(θ)

θ	0	π/2	π	3π/2	2π
r					

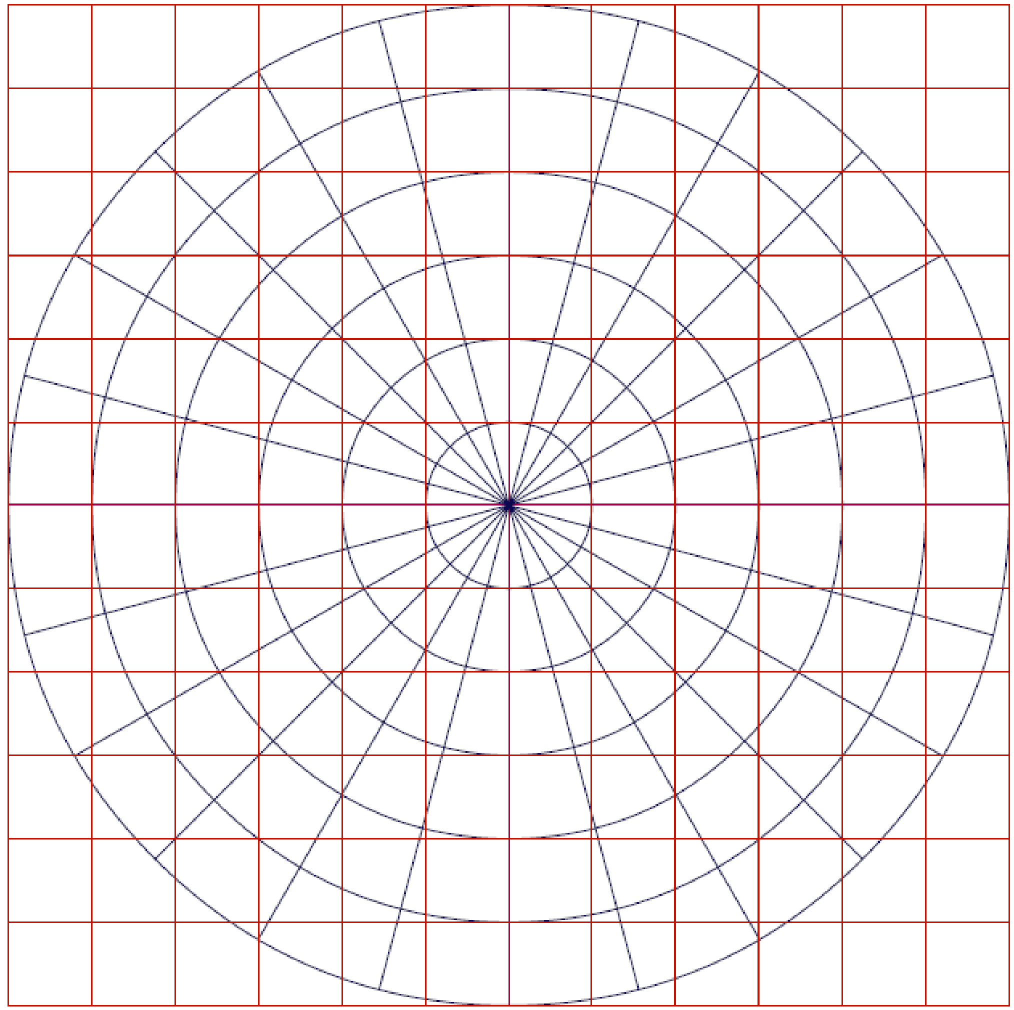
θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



Example: Graph r = cos(2θ)

θ	0	π/2	π	3π/2	2π
r					

θ	π/6	π/4	π/3	2π/3	3π/4	5π/6
r						



An old exam question:

The four polar equations below each match up with one of the six pictures. Identify which match.

1. $r = \sqrt{\theta}$
2. $r = 1 - 2\cos(\theta)$
3. $r = 1 + \sin(2\theta)$
4. $r = 9\cos(\theta)$